Spring 2012

EE 330

ENGINEERING ELECTROMAGNETICS

HW 9: *Due Friday 23 March*

5.15, 6.5, 6.10, 6.18, 6.22, 6.26, 7.1, 7.3, 7.7, 7.9, 7.12

Problem 5.15 A circular loop of radius a carrying current I_1 is located in the x–y plane as shown in Fig. P5.15. In addition, an infinitely long wire carrying current I_2 in a direction parallel with the z-axis is located at $y = y_0$.

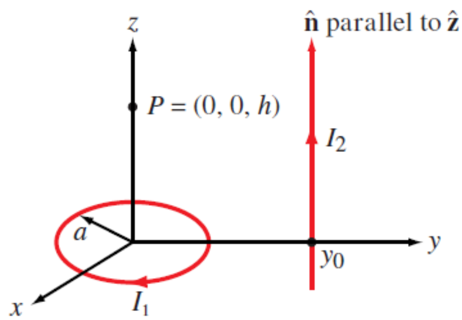

Figure P5.15: Problem 5.15.

(a) Determine **H** at $P = (0,0,h)$.

(b) Evaluate **H** for $a = 3$ cm, $y_0 = 10$ cm, $h = 4$ cm, $I_1 = 10$ A, and $I_2 = 20$ A.

Solution:

(a) The magnetic field at $P = (0,0,h)$ is composed of $\mathbf{H}_1$ due to the loop and $\mathbf{H}_2$ due to the wire:

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2.$$

From (5.34), with $z = h$,

$$\mathbf{H}_1 = \hat{\mathbf{z}}\,\frac{I_1 a^2}{2(a^2+h^2)^{3/2}} \quad \text{(A/m)}.$$

From (5.30), the field due to the wire at a distance $r = y_0$ is

$$\mathbf{H}_2 = \hat{\boldsymbol{\phi}}\,\frac{I_2}{2\pi y_0}$$

where $\hat{\boldsymbol{\phi}}$ is defined with respect to the coordinate system of the wire. Point P is located at an angel $\phi = -90°$ with respect to the wire coordinates. From Table 3-2,

$$\begin{aligned}\hat{\boldsymbol{\phi}} &= -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi \\ &= \hat{\mathbf{x}} \qquad (\text{at } \phi = -90°).\end{aligned}$$

Hence,

$$\mathbf{H} = \hat{\mathbf{z}}\,\frac{I_1 a^2}{2(a^2+h^2)^{3/2}} + \hat{\mathbf{x}}\,\frac{I_2}{2\pi y_0} \quad \text{(A/m)}.$$

(b)

$$\mathbf{H} = \hat{\mathbf{z}}\,36 + \hat{\mathbf{x}}\,31.83 \quad \text{(A/m)}.$$

Problem 6.5 A circular-loop TV antenna with 0.02-m^2 area is in the presence of a uniform-amplitude 300-MHz signal. When oriented for maximum response, the loop develops an emf with a peak value of 30 (mV). What is the peak magnitude of **B** of the incident wave?

Solution: TV loop antennas have one turn. At maximum orientation, Eq. (6.5) evaluates to $\Phi = \int \mathbf{B} \cdot d\mathbf{s} = \pm BA$ for a loop of area A and a uniform magnetic field with magnitude $B = |\mathbf{B}|$. Since we know the frequency of the field is $f = 300$ MHz, we can express B as $B = B_0 \cos(\omega t + \alpha_0)$ with $\omega = 2\pi \times 300 \times 10^6$ rad/s and α_0 an arbitrary reference phase. From Eq. (6.6),

$$V_{\text{emf}} = -N\frac{d\Phi}{dt} = -A\frac{d}{dt}[B_0 \cos(\omega t + \alpha_0)] = AB_0\omega \sin(\omega t + \alpha_0).$$

V_{emf} is maximum when $\sin(\omega t + \alpha_0) = 1$. Hence,

$$30 \times 10^{-3} = AB_0\omega = 0.02 \times B_0 \times 6\pi \times 10^8,$$

which yields $B_0 = 0.8$ (nA/m).

Problem 6.10 A 50-cm-long metal rod rotates about the z-axis at 90 revolutions per minute, with end 1 fixed at the origin as shown in Fig. P6.10. Determine the induced emf V_{12} if $\mathbf{B} = \hat{\mathbf{z}}2 \times 10^{-4}$ T.

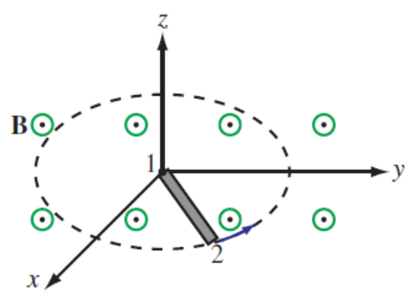

Figure P6.10: Rotating rod of Problem 6.10.

Solution: Since **B** is constant, $V_{\text{emf}} = V^{\text{m}}_{\text{emf}}$. The velocity **u** for any point on the bar is given by $\mathbf{u} = \hat{\boldsymbol{\phi}} r\omega$, where

$$\omega = \frac{2\pi \text{ rad/cycle} \times (90 \text{ cycles/min})}{(60 \text{ s/min})} = 3\pi \text{ rad/s}.$$

From Eq. (6.24),

$$\begin{aligned} V_{12} = V^{\text{m}}_{\text{emf}} = \int_2^1 (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} &= \int_{r=0.5}^{0} (\hat{\boldsymbol{\phi}} 3\pi r \times \hat{\mathbf{z}}2 \times 10^{-4}) \cdot \hat{\mathbf{r}}\, dr \\ &= 6\pi \times 10^{-4} \int_{r=0.5}^{0} r\, dr \\ &= 3\pi \times 10^{-4} r^2 \Big|_{0.5}^{0} \\ &= -3\pi \times 10^{-4} \times 0.25 = -236 \quad (\mu\text{V}). \end{aligned}$$

Problem 6.18 An electromagnetic wave propagating in seawater has an electric field with a time variation given by $\mathbf{E} = \hat{\mathbf{z}} E_0 \cos \omega t$. If the permittivity of water is $81\varepsilon_0$ and its conductivity is 4 (S/m), find the ratio of the magnitudes of the conduction current density to displacement current density at each of the following frequencies:

(a) 1 kHz

(b) 1 MHz

(c) 1 GHz

(d) 100 GHz

Solution: From Eq. (6.44), the displacement current density is given by

$$\mathbf{J}_\mathrm{d} = \frac{\partial}{\partial t}\mathbf{D} = \varepsilon \frac{\partial}{\partial t}\mathbf{E}$$

and, from Eq. (4.67), the conduction current is $\mathbf{J} = \sigma \mathbf{E}$. Converting to phasors and taking the ratio of the magnitudes,

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_\mathrm{d}}\right| = \left|\frac{\sigma \widetilde{\mathbf{E}}}{j\omega\varepsilon_r\varepsilon_0\widetilde{\mathbf{E}}}\right| = \frac{\sigma}{\omega\varepsilon_r\varepsilon_0}.$$

(a) At $f = 1$ kHz, $\omega = 2\pi \times 10^3$ rad/s, and

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_\mathrm{d}}\right| = \frac{4}{2\pi \times 10^3 \times 81 \times 8.854 \times 10^{-12}} = 888 \times 10^3.$$

The displacement current is negligible.

(b) At $f = 1$ MHz, $\omega = 2\pi \times 10^6$ rad/s, and

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_\mathrm{d}}\right| = \frac{4}{2\pi \times 10^6 \times 81 \times 8.854 \times 10^{-12}} = 888.$$

The displacement current is practically negligible.

(c) At $f = 1$ GHz, $\omega = 2\pi \times 10^9$ rad/s, and

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_\mathrm{d}}\right| = \frac{4}{2\pi \times 10^9 \times 81 \times 8.854 \times 10^{-12}} = 0.888.$$

Neither the displacement current nor the conduction current are negligible.

(d) At $f = 100$ GHz, $\omega = 2\pi \times 10^{11}$ rad/s, and

$$\left|\frac{\widetilde{\mathbf{J}}}{\widetilde{\mathbf{J}}_\mathrm{d}}\right| = \frac{4}{2\pi \times 10^{11} \times 81 \times 8.854 \times 10^{-12}} = 8.88 \times 10^{-3}.$$

The conduction current is practically negligible.

Problem 6.22 If we were to characterize how good a material is as an insulator by its resistance to dissipating charge, which of the following two materials is the better insulator?

$$\begin{array}{lll} \text{Dry Soil:} & \varepsilon_r = 2.5, & \sigma = 10^{-4} \text{ (S/m)} \\ \text{Fresh Water:} & \varepsilon_r = 80, & \sigma = 10^{-3} \text{ (S/m)} \end{array}$$

Solution: Relaxation time constant $\tau_r = \frac{\varepsilon}{\sigma}$.

$$\text{For dry soil,} \qquad \tau_r = \frac{2.5}{10^{-4}} = 2.5 \times 10^4 \text{ s.}$$

$$\text{For fresh water,} \qquad \tau_r = \frac{80}{10^{-3}} = 8 \times 10^4 \text{ s.}$$

Since it takes longer for charge to dissipate in fresh water, it is a better insulator than dry soil.

Problem 6.26 The electric field radiated by a short dipole antenna is given in spherical coordinates by

$$\mathbf{E}(R, \theta; t) = \hat{\boldsymbol{\theta}} \, \frac{2 \times 10^{-2}}{R} \sin\theta \; \cos(6\pi \times 10^8 t - 2\pi R) \quad \text{(V/m)}.$$

Find $\mathbf{H}(R, \theta; t)$.

Solution: Converting to phasor form, the electric field is given by

$$\widetilde{\mathbf{E}}(R, \theta) = \hat{\boldsymbol{\theta}} E_\theta = \hat{\boldsymbol{\theta}} \frac{2 \times 10^{-2}}{R} \sin\theta e^{-j2\pi R} \quad \text{(V/m)},$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned} \widetilde{\mathbf{H}}(R, \theta) = \frac{1}{-j\omega\mu} \nabla \times \widetilde{\mathbf{E}} &= \frac{1}{-j\omega\mu} \left[\hat{\mathbf{R}} \frac{1}{R\sin\theta} \frac{\partial E_\theta}{\partial \phi} + \hat{\boldsymbol{\phi}} \frac{1}{R} \frac{\partial}{\partial R} (R E_\theta) \right] \\ &= \frac{1}{-j\omega\mu} \hat{\boldsymbol{\phi}} \, \frac{2 \times 10^{-2}}{R} \sin\theta \, \frac{\partial}{\partial R} (e^{-j2\pi R}) \\ &= \hat{\boldsymbol{\phi}} \frac{2\pi}{6\pi \times 10^8 \times 4\pi \times 10^{-7}} \frac{2 \times 10^{-2}}{R} \sin\theta e^{-j2\pi R} \\ &= \hat{\boldsymbol{\phi}} \, \frac{53}{R} \sin\theta \; e^{-j2\pi R} \quad (\mu\text{A/m}). \end{aligned}$$

Converting back to instantaneous value, this is

$$\mathbf{H}(R, \theta; t) = \hat{\boldsymbol{\phi}} \, \frac{53}{R} \sin\theta \cos(6\pi \times 10^8 t - 2\pi R) \quad (\mu\text{A/m}).$$

Problem 7.1 The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{z}}\,30\cos(10^8 t - 0.5y) \quad \text{(mA/m)}$$

Find the following:

(a) The direction of wave propagation.

(b) The phase velocity.

(c) The wavelength in the material.

(d) The relative permittivity of the material.

(e) The electric field phasor.

Solution:

(a) Positive y-direction.

(b) $\omega = 10^8$ rad/s, $k = 0.5$ rad/m.

$$u_\mathrm{p} = \frac{\omega}{k} = \frac{10^8}{0.5} = 2 \times 10^8 \text{ m/s.}$$

(c) $\lambda = 2\pi/k = 2\pi/0.5 = 12.6$ m.

(d) $\varepsilon_\mathrm{r} = \left(\dfrac{c}{u_\mathrm{p}}\right)^2 = \left(\dfrac{3 \times 10^8}{2 \times 10^8}\right)^2 = 2.25.$

(e) From Eq. (7.39b),

$$\begin{aligned}
\widetilde{\mathbf{E}} &= -\eta\hat{\mathbf{k}} \times \widetilde{\mathbf{H}}, \\
\eta &= \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_\mathrm{r}}} = \frac{120\pi}{1.5} = 251.33 \quad (\Omega), \\
\hat{\mathbf{k}} &= \hat{\mathbf{y}}, \quad \text{and} \quad \widetilde{\mathbf{H}} = \hat{\mathbf{z}}30e^{-j0.5y} \times 10^{-3} \quad \text{(A/m).}
\end{aligned}$$

Hence,

$$\widetilde{\mathbf{E}} = -251.33\hat{\mathbf{y}} \times \hat{\mathbf{z}}30e^{-j0.5y} \times 10^{-3} = -\hat{\mathbf{x}}7.54e^{-j0.5y} \quad \text{(V/m),}$$

and

$$\mathbf{E}(y,t) = \Re\mathfrak{e}(\widetilde{\mathbf{E}}e^{j\omega t}) = -\hat{\mathbf{x}}7.54\cos(10^8 t - 0.5y) \quad \text{(V/m).}$$

Problem 7.3 The electric field phasor of a uniform plane wave is given by $\widetilde{\mathbf{E}} = \hat{\mathbf{y}}\,10e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5×10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find the following:

(a) The wavelength.

(b) The frequency f of the wave.

(c) The relative permittivity of the medium.

(d) The magnetic field $\mathbf{H}(z,t)$.

Solution:

(a) From $\widetilde{\mathbf{E}} = \hat{\mathbf{y}}10e^{j0.2z}$ (V/m), we deduce that $k = 0.2$ rad/m. Hence,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2} = 10\pi = 31.42 \text{ m.}$$

(b)

$$f = \frac{u_p}{\lambda} = \frac{1.5 \times 10^8}{31.42} = 4.77 \times 10^6 \text{ Hz} = 4.77 \text{ MHz.}$$

(c) From

$$u_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}}, \qquad \varepsilon_r = \frac{1}{\mu_r}\left(\frac{c}{u_p}\right)^2 = \frac{1}{2.4}\left(\frac{3}{1.5}\right)^2 = 1.67.$$

(d)

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \simeq 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{2.4}{1.67}} = 451.94 \quad (\Omega),$$

$$\widetilde{\mathbf{H}} = \frac{1}{\eta}(-\hat{\mathbf{z}}) \times \widetilde{\mathbf{E}} = \frac{1}{\eta}(-\hat{\mathbf{z}}) \times \hat{\mathbf{y}}10e^{j0.2z} = \hat{\mathbf{x}}22.13e^{j0.2z} \quad \text{(mA/m)},$$

$$\mathbf{H}(z,t) = \hat{\mathbf{x}}22.13\cos(\omega t + 0.2z) \quad \text{(mA/m)},$$

with $\omega = 2\pi f = 9.54\pi \times 10^6$ rad/s.

Problem 7.7 A 60-MHz plane wave traveling in the $-x$-direction in dry soil with relative permittivity $\varepsilon_r = 4$ has an electric field polarized along the z-direction. Assuming dry soil to be approximately lossless, and given that the magnetic field has a peak value of 10 (mA/m) and that its value was measured to be 7 (mA/m) at $t = 0$ and $x = -0.75$ m, develop complete expressions for the wave's electric and magnetic fields.

Solution: For $f = 60$ MHz $= 6 \times 10^7$ Hz, $\varepsilon_r = 4$, $\mu_r = 1$,

$$k = \frac{\omega}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 6 \times 10^7}{3 \times 10^8}\sqrt{4} = 0.8\pi \quad \text{(rad/m)}.$$

Given that $\mathbf{E}$ points along $\hat{\mathbf{z}}$ and wave travel is along $-\hat{\mathbf{x}}$, we can write

$$\mathbf{E}(x,t) = \hat{\mathbf{z}} E_0 \cos(2\pi \times 60 \times 10^6 t + 0.8\pi x + \phi_0) \quad \text{(V/m)}$$

where E_0 and ϕ_0 are unknown constants at this time. The intrinsic impedance of the medium is

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{2} = 60\pi \quad (\Omega).$$

With $\mathbf{E}$ along $\hat{\mathbf{z}}$ and $\hat{\mathbf{k}}$ along $-\hat{\mathbf{x}}$, (7.39) gives

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E}$$

or

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\frac{E_0}{\eta}\cos(1.2\pi \times 10^8 t + 0.8\pi x + \phi_0) \quad \text{(A/m)}.$$

Hence,

$$\frac{E_0}{\eta} = 10 \quad \text{(mA/m)}$$
$$E_0 = 10 \times 60\pi \times 10^{-3} = 0.6\pi \quad \text{(V/m)}.$$

Also,

$$H(-0.75 \text{ m}, 0) = 7 \times 10^{-3} = 10\cos(-0.8\pi \times 0.75 + \phi_0) \times 10^{-3}$$

which leads to $\phi_0 = 153.6^\circ$.

Hence,

$$\mathbf{E}(x,t) = \hat{\mathbf{z}}\, 0.6\pi \cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6^\circ) \quad \text{(V/m)}.$$
$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\, 10\cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6^\circ) \quad \text{(mA/m)}.$$

Problem 7.9 For a wave characterized by the electric field

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\, a_x \cos(\omega t - kz) + \hat{\mathbf{y}}\, a_y \cos(\omega t - kz + \delta)$$

identify the polarization state, determine the polarization angles (γ, χ), and sketch the locus of $\mathbf{E}(0,t)$ for each of the following cases:

(a) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 0$

(b) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 180°$

(c) $a_x = 3$ V/m, $a_y = 3$ V/m, and $\delta = 45°$

(d) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = -135°$

Solution:

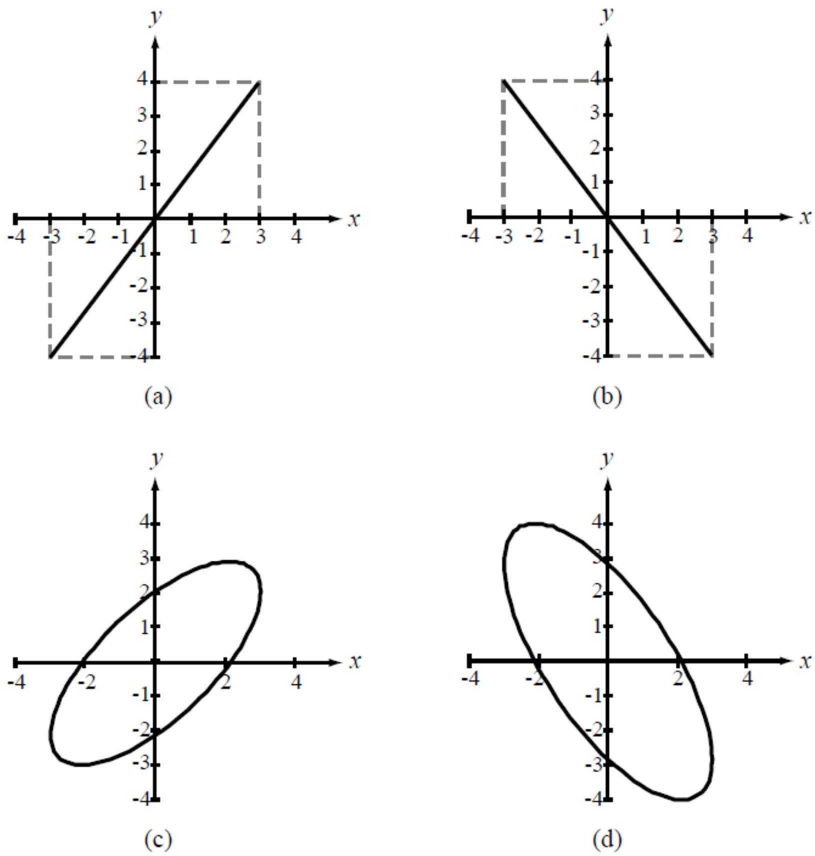

Figure P7.9: Plots of the locus of $\mathbf{E}(0,t)$.

$$\psi_0 = \tan^{-1}(a_y/a_x), \quad \text{[Eq. (7.60)]},$$
$$\tan 2\gamma = (\tan 2\psi_0)\cos\delta \quad \text{[Eq. (7.59a)]},$$
$$\sin 2\chi = (\sin 2\psi_0)\sin\delta \quad \text{[Eq. (7.59b)]}.$$

Case	a_x	a_y	δ	ψ_0	γ	χ	Polarization State
(a)	3	4	0	53.13°	53.13°	0	Linear
(b)	3	4	180°	53.13°	−53.13°	0	Linear
(c)	3	3	45°	45°	45°	22.5°	Left elliptical
(d)	3	4	−135°	53.13°	−56.2°	−21.37°	Right elliptical

(a) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}4\cos(\omega t - kz)$.

(b) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) - \hat{\mathbf{y}}4\cos(\omega t - kz)$.

(c) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}3\cos(\omega t - kz + 45°)$.

(d) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}4\cos(\omega t - kz - 135°)$.

Problem 7.12 The electric field of an elliptically polarized plane wave is given by

$$\mathbf{E}(z,t) = [-\hat{\mathbf{x}}\,10\sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}}\,30\cos(\omega t - kz)] \quad \text{(V/m)}$$

Determine the following:

(a) The polarization angles (γ, χ).

(b) The direction of rotation.

Solution:

(a)

$$\begin{aligned}\mathbf{E}(z,t) &= [-\hat{\mathbf{x}}10\sin(\omega t - kz - 60^\circ) + \hat{\mathbf{y}}30\cos(\omega t - kz)] \\ &= [\hat{\mathbf{x}}10\cos(\omega t - kz + 30^\circ) + \hat{\mathbf{y}}30\cos(\omega t - kz)] \quad \text{(V/m)}.\end{aligned}$$

Phasor form:

$$\widetilde{\mathbf{E}} = (\hat{\mathbf{x}}10e^{j30^\circ} + \hat{\mathbf{y}}30)e^{-jkz}.$$

Since δ is defined as the phase of E_y relative to that of E_x,

$$\begin{aligned}\delta &= -30^\circ, \\ \psi_0 &= \tan^{-1}\left(\frac{30}{10}\right) = 71.56^\circ, \\ \tan 2\gamma &= (\tan 2\psi_0)\cos\delta = -0.65 \quad \text{or} \quad \gamma = 73.5^\circ, \\ \sin 2\chi &= (\sin 2\psi_0)\sin\delta = -0.40 \quad \text{or} \quad \chi = -8.73^\circ.\end{aligned}$$

(b) Since $\chi < 0$, the wave is right-hand elliptically polarized.